Figure 3 shows the dependence of the path on the time for the shock wave 2 and the piston 1, as well as experimental curves of the displacement of the center of gravity of the current 4 and the front of the luminescence 3 with a pressure of 760 and 400 mm Hg (Fig. 3a and b, repsectively) [5]. It can be seen that the pressure of the current layer and the front of the luminescence, which can be connected with the magnetic piston and the shock wave, are in qualitative agreement with calculation; however, at the initial moment, the piston does not capture the gas completely and, for this reason, the experimental curves lie above the calculated. A shell is obviously formed at a distance of around 1 cm from the point of the breakdown, which is in agreement with the above-mentioned evaluations of the formation time of the piston. Therefore, lines 5 and 6 show experimental curves with a shift of the point of breakdown downward by 1 cm with respect to the point of formation of the shell. With lower pressures (30-100 mm Hg), the discharge starts in the conical part of a nozzle of variable cross section, and there is ablation of the insulator; therefore, a comparison between calculation and experiment is not justified here, since in this case the constant of the magnetic pressure depends on the time.

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DEVELOPMENT OF DYNAMIC FORMS OF BUCKLING OF ELASTOPLASTIC BEAMS WITH INTENSIVE LOADING

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1. We consider an I-beam. At the initial moment of time, an intensive longitudinal constant load is suddently applied to the beam; in the theoretical analysis, longitudinal vibrations are not taken into consideration. The intensive compressive loading is considerably greater than an Euler loading [1]. We assume that this compressive load corresponds to stresses exceeding the elastic limit. It is assumed that the bending takes place in the plane of the web, while the bending moment is taken up only by the flanges of the I-beam. A study is made of the development with time of the forms of inelastic buckling of beams with small normal bends w. Equating the sum of the internal forces with respect to the neutral line to the external moment, we find the equation of the curved axis of the I-beam [2]

$$TIw_{xxxx} + Nw_{xx} + \rho Sw_{tt} = -N(w_{0xx} + w_{1xx}). \tag{1.1}$$

Since, before loading, the freely supported beam was at rest, the initial and boundary conditions have the form

$$\begin{aligned} w &= w_t = 0,\ t = 0,\ 0 \leqslant x \leqslant L; \\ w &= w_{xx} = 0,\ x = 0,\ L,\ t \geqslant 0, \end{aligned} \tag{1.2}$$

where w is the additional normal deflection; x and t are the longitudinal coordinate and the time; N is an intensive longitudinal load; $T = 2E_1E_2/(E_1 + E_2)$ the modulus of elasticity and relief; E_2 is the reduction modulus (Fig. 1); $E = E_1$ is the tangential modulus; w_0 and w_1 are the initial regularities of the beam and the shift of the central line for cross sections of the beam; S and I are the constant area and moment of inertia of a transverse cross section; L is the length of the beam.

The function $w_1(x)$, characterizing the shift of the central line, is subject to determination. With determination of w_1, use is made of an idealized $\sigma - \epsilon$ diagram, Fig. 1, and the assumption N = const. Figure 2a shows an I-beam; the flanges of the I-beam are connected by a thin web. Three cases of the loading of beams at the initial moment of time (t = 0) are considered: 1) the beam remains elastic; 2) the stresses in the beam considerably exceed the elastic limit (point c on the idealized diagram of Fig. 1); 3) the stresses coincide with the elastic limit (point b on the idealized diagram). The

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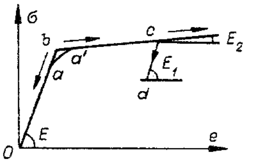

Fig. 1

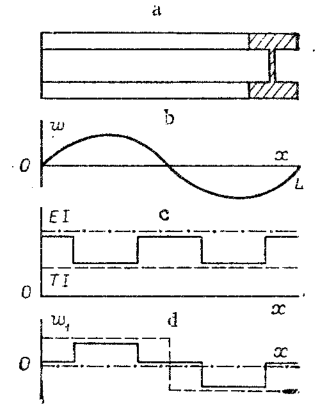

Fig. 2

distribution of the rigidities in Fig. 2c and the shift in the neutral line in Fig. 2d are shown by the dash–dot, dashed, and continuous curves, respectively, for the first, second, and third cases (the notation is analogous in Fig. 3).

The maximum of the function of the shift of the neutral line is found from the equality to zero of the resultant bending stresses (Fig. 2d)

$$\max|w_1| = h(E_1 - E_2)/2(E_1 + E_2), \tag{1.3}$$

where h is the distance between the centers of gravity of the flanges of the beam. The shift in w_1 is determined by the relationship between the modulus of the unloading and the tangential modulus. For real beams, made of structural steels, the value of the initial shift is such that $\max|w_1| \gg \max|w_0|$, where the beams were obtained by the machining of billets.

The problem obtained (1.1), (1.2), taking account of relationship (1.3) almost coincides with the analogous problem for an elastic beam [1]. An analysis is then made of motions in systems with one degree of freedom; the rise in the amplitude of the deflections in simplified systems is shown in Fig. 3. This degree of freedom corresponds to a form with the number m_*. The determination of the number m_* differs in no way from [1], since the equation of the cited work and Eq. (1.1) have constant coefficients. Thus, we have $w(x, t) = q(t) \sin m_*\pi x/L$. The approximation of the starting system with one degree of freedom is in agreement with the experimental results given below, where there is a simple loading program.

After appropriate transformations, the problem (1.1), (1.2) is reduced to the following simple problem for q(t):

$$q'' - \alpha^2 q = F, \quad q = q' = 0 \quad (t = 0),$$

$$\alpha^2 = \frac{\pi^4 TI}{\rho S L^4} m_*^2 (\eta^2 - m_*^2), \quad \eta^2 = \frac{N}{N_e}, \quad N_e = \frac{\pi^2 TI}{L^2}, \quad F = c_1^{(0)} + c_1^{(1)},$$

where η is the parameter of the intensity of the loading; N_e is an Euler load; $c_1^{(0)}$ and $c_1^{(1)}$ are coefficients, corresponding to the Fourier coefficient of the functions w_0 and w_1. The number m_* corresponds to motion describing the buckling of an elastoplastic beam; $q = F\alpha^{-2}(\operatorname{ch}\alpha t - 1)$. Finally, for additional bending, we have $w = F\alpha^{-2}(\operatorname{ch}\alpha t - 1)\sin m_*\pi x/L$. We note that the rate of growth of the amplitude of the additional bending in the inelastic case is considerably greater than in the elastic case, since $|c_1^{(1)}| \gg |c_1^{(0)}|$.

At the moment of time $t = t_1$, the constant compressive force is removed. Let the beam then be deformed elastically. The behavior of the beam in the period of time $t > t_1$ is described by Eq. (1.1) with N = 0 and $E = E_1 = E_2$ ($w_1 = 0$). As before we assume that the form of the additional bending coincided with a determined form of buckling. The motion of the beam with $t > t_1$ becomes vibrational $q_1 = A\cos\alpha_1(t - t_1) + B\sin\alpha_1(t - t_1)$. For the additional bending w_2 we obtain

$$w_2 = [F\alpha^{-2}(\operatorname{ch}\alpha t_1 - 1)\cos\alpha_1(t - t_1) + F\alpha^{-1}\operatorname{sh}\alpha t_1 \sin\alpha_1(t - t_1)]\sin(m_*\pi x/L). \tag{1.4}$$

We note that the expression constructed above (1.4) for the additional bending w_2 describes the behavior of an elastoplastic beam as a system with one degree of freedom, and is an evaluation close to an upper evaluation of the value of the bending.

2. Intensive loading of the beam was carried out in a special unit, a scheme of which is shown in Fig. 4. Isolation of the plane of the deformation of the beam 1 was assured by the side walls 2 of the assembly. The time of action of the pressure of the explosion products with the detonation of an explosive 3 is usually small; to increase this time, a layer of

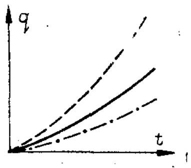

Fig. 3

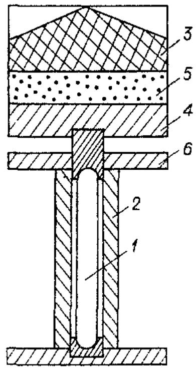

Fig. 4

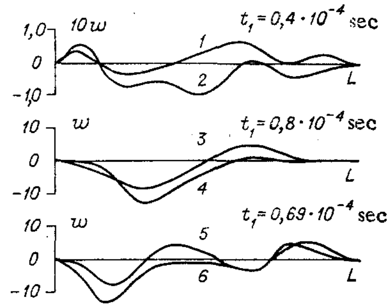

Fig. 5

porous metallic material 5 is placed between the charge of explosive and the striker 4. The exposive was ammonite 6ZhV. The value of the displacement of the striker was varied by a change in the starting distance between the striker and the intersecting plate 6.

Two series of experiments were made; in the first series, the weighed portion of explosive and the thickness of the porous material were selected experimentally. In the second series of experiments, the height of the layer of explosive (without a generator of a plane shock wave) was 50 mm, and the thickness of the layer (80 mm) of metallic porous material assured a duration of the action of a load of ~35 tons on the firing pin of the order of 10^{-5} sec (more exactly from 10^{-5} to 10^{-4} sec), taking account of the ratio of the masses of the striker and the compressed beam. In the second series, the longitudinal shifts of the unit in the first four experiments differ from the last two, since in the first experiments the unit with the beam was mounted on a table of metallic shot, while, in the fifth and sixth experiments, the unit rested on a solid steel table. By the same token, from the first to the fourth experiments, a simple loading program was effected: with $0 \leqslant t \leqslant t_1$ the load is close to constant, and is then removed. In the two last experiments, the program was more complex; with the first passage of an inelastic wave, the load is close to constant, and, with reflection from the support, the longitudinal load rises due to the great rigidity of the table (in the calculations it was assumed that, after reflection, the load increased by 1.75 times [3]). In the second series of experiments, partial breakdown of the samples was observed.

The I-beams for the tests were made from an All-Union State Standard (GOST) 2591-71 square rod with limiting deviations of its dimensions of 0.5 mm, followed by milling of the channels. The length of the sample L = 200 mm, the side of the square 10 mm, and width of a channel 4 mm, and the thickness of the thin wall 1.5 mm. The material of the rod was steel 9KhS GOST 5950-63 in the delivered condition. The dependence of the stresses on the deformation with compression for samples of the above material is shown in Fig. 1. The elastic modulus E_1 and the tangential modulus E_2 are equal to $E_1 = 2.35 \cdot 10^6$ kg/cm², $E_2 = 7.4 \cdot 10^4$ kg/cm² [4].

Figure 5 shows the projection of the profile of a beam after tests in the plane of the bending; the loading end is on the left. The samples were measured in a universal UIM-21 instrumental microscope; the exactness of the measurements was one micron. To elucidate the validity of the postulation that the behavior of a complex inelastic system can be described by the motion of a system with one degree of freedom, the residual deflections of the beams were analyzed. These deflections were represented in the form of Fourier series. The results of a harmonic analysis of the experimental results are given in Table 1, where, in the first row there are shown the numbers of the beams, while, in a vertical direction there are arranged the first four coefficients of the Fourier series for the six samples tested.

We recall that, with the intensive loading of elastic beams in a finite interval of time [5], either one or two forms of the motion can be distinguished, where this interval is not very small. In Section 1 simple relationships were obtained for the residual deflections of inelastic beams with one determined form of motion.

In the experiments (see Table 1) for samples 2, 3, 5, this corresponds to the first, second, and fourth forms. For samples 1, 4, 6, there are two determining forms of motion, respectively: second and third, first and second, second and fourth. From Table 1 it can be seen that the determining form of motion with elastoplastic buckling is the second form $m_* = 2$ for the first four samples.

The residual deflections are evaluated by the formula

$$q = c_1^{(1)} \alpha^{-2} (\operatorname{ch} \alpha t_1 - 1), \tag{2.1}$$

where the constant $c_1^{(1)}$ corresponds to a shift in the central axis in an inelastic beam $\max |w_1| = 3.3$ mm. The measured initial irregularities before the tests of the beams are such that $\max |w_0| = 0.1$ mm. We note that for elastic strain or for the inelastic strain, if we follow the method of [2], the constant in Eq. (2.1) is substituted for $c_1^{(0)}$ $(|c_1^{(1)}| \gg c_1^{(0)})$.

The loading time in the experiments was regulated by the value of the displacement l of the striker, and was roughly calculated by the formulas

TABLE 1

	1	2	3	4	5	6
$q_1(t_1)$	0,11	—0,54	—1,28	—4,65	—0,14	—1,89
$q_2(t_1)$	—0,22	0,02	4,48	4,56	0,9	3,13
$q_3(t_1)$	0,15	0,22	—0,94	—0,35	—0,89	—1,52
$q_4(t_1)$	0,19	0,19	—2,6	—2,45	3,4	3,17

TABLE 2

$t_1 \cdot 10^4$, sec	$\|q_{m_*}\|$			$\sum_{m=1}^{4} \|q_m\|$
	1	2	3	
$\frac{0,1}{0,4}$	$\frac{0,11}{1,97}$	$\frac{0,002}{0,06}$	0,22	0,67; 0,97
$\frac{0,2}{0,8}$	$\frac{0,46}{10,2}$	$\frac{0,009}{0,3}$	4,48	9,3; 12,0
$\frac{0,17}{0,69}$	$\frac{0,18}{8,86}$	$\frac{0,005}{0,27}$	3,13	5,33; 9,71

$$t_1 = l(T\rho)^{1/2}\sigma^{-1}; \quad (2.2)$$
$$t_1 = l(E\rho)^{1/2}\sigma^{-1}, \quad (2.3)$$

where σ is the compressive stress in the beam. Table 2 (columns 1, 2) gives evaluation of the amplitudes of the determining form of motion in accordance with the proposed method and the method of [2]. Column 3 gives the amplitudes of the buckling of the first, third, and sixth beams. In Table 2 the numerator is the loading time calculated using formula (2.2) and the corresponding amplitude (2.1); the denominator gives analogous characteristics, calculated in accordance with formulas (2.3), (2.1).

In experiments on samples 5-6, the unit was mounted on a rigid baseplate. According to the data of [3], the stresses in a beam increase by 1.75 times on the average, which led to the breakdown of sample 6. A theoretical determining form of the motion ($m_* = 2$, $m_* = 3$) in experiments on samples 5, 6, could not be achieved, due to the complex loading program. The last column in Table 2 gives upper evaluations for the amplitudes of the deflections, respectively, of the first and second, third and fourth, fifth and sixth beams.

As we can see, the evaluation obtained for the value of the residual deflection gives satisfactory agreement with experiment. In samples 1 and 2, the initial state of the motion was recorded, the forms of the buckling are equally correct, and the deflections are small. We note the great sensitivity of the process of the buckling of a beam to a change in the time of the action of the load. An increase in the loading time by 2 times leads to an increase in the deflections by an order of magnitude.

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